

Marginal Deformations as Lower Dimensional D–brane Solutions in Open String Field theory

Bum-Hoon Lee

*Center for Quantum SpaceTime (CQUeST) and Department of Physics,
Sogang University, Shinsu-dong 1, Mapo-gu, Seoul, Korea
E-mail: bhl@sogang.ac.kr*

Chanyong Park

*Center for Quantum SpaceTime (CQUeEST), Sogang University
Shinsu-dong 1, Mapo-gu, Seoul, Korea
E-mail: cyong21@sogang.ac.kr*

D.D.Tolla

*Center for Quantum SpaceTime (CQUeST), Sogang University
Shinsu-dong 1, Mapo-gu, Seoul, Korea
E-mail: tolla@sogang.ac.kr*

ABSTRACT: By direct calculation we showed that a finite analytic solution for marginal deformation of open string field theory, by a matter primary operator with singular OPE, can be obtained to all orders in the deformation parameter. In particular, we obtained solutions that describe lower dimensional D–branes and our results agree with the results obtained when the same problem is treated in the world-sheet conformal field theory language.

Contents

1. Introduction	1
2. The action of B/L and the OPE of V	2
3. The tachyon profile	11
4. Conclusion	17

1. Introduction

One point of view of understanding D-branes is that they are solutions of string field theory equation of motion. Different solutions of string field theory equation of motion represent different two dimensional conformal field theory (CFT) backgrounds. Inspired by the Schnabl's analytic construction of open string field theory (OSFT) equation of motion representing the tachyon vacuum [1] (see [2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13] for more development on this), recently several more solutions have been obtained [14, 15, 16, 18, 19, 20, 22] for both bosonic and supersymmetric OSFT. These new solutions describe conformal field theories that are deformed by exactly marginal operators. Among them are the solutions representing the CFT of lower-dimensional D-branes.

It is a well established fact [23, 24, 25, 26, 27, 28] that the boundary conformal field theory (BCFT) describing a Dp -brane in bosonic string theory is identical to that of a $D(p+1)$ -brane deformed by an exactly marginal boundary operator. More precisely, one can deform the former BCFT into the later by adding an exactly marginal boundary term

$$\delta S_{ws} = \tilde{\lambda} \int dt \sqrt{2} \cos(X(t)) \quad (1.1)$$

to the world-sheet action, where X is the direction transverse to the Dp -brane, t is a coordinate on the world-sheet boundary and $\tilde{\lambda}$ is a free parameter. At ($\tilde{\lambda} = \pm \frac{1}{2}$) with this we obtain a periodic array of Dp -branes, with Dirichlet boundary condition on X , placed at $(x = (2n+1)\pi)$ if we choose the plus sign and at $(x = 2n\pi)$ if we choose the minus sign.

An alternative description of marginal deformations in the framework of string field theory was considered in [29, 30, 31, 32, 33, 34]. It was shown that, switching on a boundary marginal deformation operator give rise to a string field theory configuration corresponding to a new classical solution of the equation of motion of OSFT formulated around the original undeformed BCFT. In these investigations, mainly the level truncation method in the Siegel gauge was used and switching on the marginal boundary operator in world sheet was interpreted as giving vacuum expectation values (vev) to the fields associated

to the tachyonic and the massless open string modes. The rolling tachyon solution by Sen [35] is the best example for such description of marginal deformations in string field theory framework, where the vev was turned on only for the tachyonic mode.

A recent construction of analytic solutions for marginal deformations in OSFT [14, 15] used the recursive technique developed in [35] in a new gauge introduced by Schnabl in [1] ($B\Psi = 0$), where B is the antighost zero mode in the conformal frame of the sliver. The ansatz for the solutions were given by a series expansion in some parameter λ which to the first order can be identified with the coupling constant $\tilde{\lambda}$ of the exactly marginal operator we mentioned above. One can then solve the equation of motion at each order of λ . These techniques were very effective to obtain solutions generated by a marginal deformation operator $V(z)$ that has a regular OPE with itself. When the OPE is singular, divergences arises as the separation between boundary insertions approaches zero and one needs to add counter terms at each order of λ to regularize it. However, the form of the counter terms were obtained only up to the third order by a clever guess and their forms for higher order terms are not known. One purpose of this paper is to study the origin of the divergences in the case of marginal deformations with singular OPE and to develop a method to determine the counter terms necessary to cancel the divergences at any order. In an earlier work [15] it was mentioned as an open problem that some of the counter terms violate the gauge condition even though the solutions were constructed to respect the gauge. In this paper we will demonstrate by explicit calculations that in the case of singular OPE marginal deformation, unlike the regular ones, only a piece of the solution can respect the Schnabl gauge and it is not surprising to have counter terms outside the gauge.

The rest of the paper is organized as follows. In section 2 we will consider solutions with both regular OPE and singular OPE. We will show that the main difference between these solutions is the fact that the first one can be expanded only in terms of states with positive L eigenvalue, where L is the Virasoro operator L_0 in the conformal frame of the sliver, while the second one contains the eigenvalues 0 and -1 as well. In the same section we will show that the divergences in the case of singular OPE arise from inverting the L operator on zero eigenvalue states and using the Schwinger representation of (L^{-1}) on negative eigenvalue states. Knowing the origin of the divergences we could easily determine the form of the counter terms to add at each level to regularize the solution. In section 3 we will use the procedure we developed in section 2 to write solutions representing arrays of D24-branes, which are obtained when an exactly marginal boundary deformation is turned on along the 25-th direction. In section 4 we will discuss our results.

2. The action of B/L and the OPE of V

The linearized string field theory equation of motion ($Q_B\Psi = 0$) is satisfied by the state $\Psi^{(1)} = cV(0)|0\rangle$ corresponding to the operator $cV(0)$, for any dimension one matter primary operator V . An ansatz for new class of solutions for the non-linear equation of motion

$(Q_B \Psi + \Psi * \Psi = 0)$ were made as an expansion in some parameter λ .

$$\Psi_\lambda = \sum_{n=1}^{\infty} \lambda^n \Psi^{(n)}, \quad (2.1)$$

with $\Psi^{(n)}$ satisfying

$$Q_B \Psi^{(n)} = \Phi^{(n)}, \quad n > 1 \quad (2.2)$$

where $\Phi^{(n)}$ is BRST exact and is given by

$$\Phi^{(n)} = - \sum_{k=1}^{n-1} \Psi^{(n-k)} * \Psi^{(k)} \quad (2.3)$$

If Ψ is in Schnabl gauge ($B\Psi_\lambda = 0$) and there is no overlap between $\Phi^{(n)}$ and the kernel of L the solution can be written as

$$\Psi^{(n)} = \frac{B}{L} \Phi^{(n)} \quad (2.4)$$

Further more if $\Phi^{(n)}$ does not contain states with negative L eigenvalues we can write

$$\Psi^{(n)} = \int_0^\infty dT B e^{-TL} \Phi^{(n)} \quad (2.5)$$

In this section we will show that such a solution is allowed only when $V(z)$ has a regular OPE with itself while in the case of singular OPE, only part of the solution can be written as in (2.5). Here we notice that if not for the action of L^{-1} , in 2.4 the operators are inserted at finite distances from each other along the real axis of the conformal frame of the sliver and every thing is regular, even if the matter primary operator has a singular OPE with itself. However, the action of L^{-1} deletes a strip of certain width and make the operators to collide. Therefore, the origin of any singularity is the action of L^{-1} on states of zero L eigenvalues or its Schwinger representation on states of negative L eigenvalues. We will see this in detail below.

Lets begin with the regular OPE case where

$$\lim_{z_1 \rightarrow z_2} V(z_1) V(z_2) = \text{regular} \quad (2.6)$$

Using this we can easily verify that the commutation relation for the modes of V is

$$[V_m, V_n] = \oint \frac{dz_2}{2\pi i} \text{Res}_{z_1 \rightarrow z_2} z_1^m z_2^n V(z_1) V(z_2) = 0, \quad \forall m, n \quad (2.7)$$

It is also true that for $m \geq 0$, $V_m|0\rangle = 0$, as the conformal dimension of V is one. We start our computation with the lowest level of (2.3).

$$\Phi^{(2)} = -\Psi^{(1)} * \Psi^{(1)} = -cV(0)|0\rangle * cV(0)|0\rangle \quad (2.8)$$

In the conformal frame of the sliver

$$\Phi^{(2)} = -\tilde{c}\tilde{V}(0)|0\rangle * \tilde{c}\tilde{V}(0)|0\rangle. \quad (2.9)$$

Note that as cV is a primary operator of conformal dimension zero so that there is no associated conformal factor in front. This star product can be easily be carried out as it is the simplest case of the star product of wedge states with insertions

$$U_r^\dagger U_r \tilde{\phi}_1(z_1)|0\rangle * U_s^\dagger U_s \tilde{\phi}_2(z_2)|0\rangle = U_{r+s-1}^\dagger U_{r+s-1} \tilde{\phi}_1(z_1 + \frac{s-1}{2}) \tilde{\phi}_2(z_2 - \frac{r-1}{2})|0\rangle \quad (2.10)$$

which we can write, after obvious shift of coordinate ($\tilde{z}_i \rightarrow \tilde{z}_i + \frac{r-1}{2}$), as

$$U_r^\dagger U_r \tilde{\phi}_1(z_1)|0\rangle * U_s^\dagger U_s \tilde{\phi}_2(z_2)|0\rangle = U_{r+s-1}^\dagger U_{r+s-1} \tilde{\phi}_1(z_1 + \frac{r+s-2}{2}) \tilde{\phi}_2(z_2)|0\rangle \quad (2.11)$$

where $U_r^\dagger U_r = e^{-\frac{1}{2}(r-2)L^+}$ with $L^+ = L + L^\dagger$. If we have more than two states to star multiply we use (2.10) associatively and do the appropriate shift of coordinate at the end, as the shift we have just made is not associative. In our simple case, which is ($r = s = 2$) we find

$$\Phi^{(2)} = -U_3^\dagger U_3 \tilde{c} \tilde{V}(1) \tilde{c} \tilde{V}(0) |0\rangle. \quad (2.12)$$

Expanding both $\tilde{c}(z)$ and $\tilde{V}(z)$ in their oscillator modes we can write

$$\Phi^{(2)} = -e^{-\frac{1}{2}L^+} \sum_l \sum_m \tilde{c}_l \tilde{c}_1 \tilde{V}_m \tilde{V}_{-1} |0\rangle. \quad (2.13)$$

As all commutations and anticommutations of the oscillator modes appearing in this expression are trivial the range of the indices will be

$$\Phi^{(2)} = -\sum_{r=0}^{\infty} \frac{1}{(-2)^r r!} (L^+)^r \sum_{l=-\infty}^1 \sum_{m=-\infty}^{-1} \tilde{c}_l \tilde{c}_1 \tilde{V}_m \tilde{V}_{-1} |0\rangle. \quad (2.14)$$

Here we notice that each term of this multiple sum is an eigenstate of L with eigenvalue ($l_0 = r - (l + m) \geq 1$) for (r, l, m) in the these ranges. Therefore, we conclude that if V has a regular OPE with itself, there is no overlap between the kernel of L and $\Phi^{(2)}$ does not contain any term with negative L eigenvalue. For higher order $\Phi^{(n)}$ we will have similar expression with more \tilde{V}_m, \tilde{c}_l and $B^+ = B + B^\dagger$ insertions. With l, m still in the range given above and noting that B^+ raise the L eigenvalue by one we see that higher order $\Phi^{(n)}$ also does not contain negative or zero L eigenvalues. Therefore, it is safe to invert L or use the Schwinger representation of L^{-1} on $\Phi^{(n)}$ for $\forall n > 1$ when V has regular OPE with itself.

Next lets consider the case where V has singular OPE, in particular

$$V(z_1)V(z_2) = \frac{1}{(z_1 - z_2)^2} + \text{regular}. \quad (2.15)$$

The commutation relation and the action on the vacuum of the oscillator modes will be

$$[V_m, V_n] = m\delta_{m,-n}, \quad V_l|0\rangle = 0, \quad \forall l \geq 0. \quad (2.16)$$

Therefore, unlike the case in equation (2.14) we can not drop all the positive modes of V and hence $\Phi^{(2)}$ is written as

$$\Phi^{(2)} = - \sum_{r=0}^{\infty} \frac{1}{(-2)^r r!} (L^+)^r \sum_{l=-\infty}^1 \sum_{m=-\infty}^1 \tilde{c}_l \tilde{c}_1 \tilde{V}_m \tilde{V}_{-1} |0\rangle \quad (2.17)$$

or

$$\begin{aligned} \Phi^{(2)} = & - \sum_{r=0}^{\infty} \frac{1}{(-2)^r r!} (L^+)^r \sum_{l=-\infty}^1 \sum_{m=-\infty}^{-1} \tilde{c}_l \tilde{c}_1 \tilde{V}_m \tilde{V}_{-1} |0\rangle \\ & - \sum_{r=0}^{\infty} \frac{1}{(-2)^r r!} (L^+)^r \sum_{l=-\infty}^1 \tilde{c}_l \tilde{c}_1 |0\rangle. \end{aligned} \quad (2.18)$$

The first line is exactly what we have in the case of regular OPE and hence there is no $(l_0 \leq 0)$ state in the first line. The L eigenvalue of each term in the second line is $(l_0 = r - (l + 1) \geq -1)$. Therefore, in this case there is an overlap between the kernel of L and $\Phi^{(2)}$ and it contains negative L eigenvalue terms as well. The only choices which give $(l_0 = 0)$ are

$$(r = 0, l = -1), \quad (r = 1, l = 0) \quad (2.19)$$

and the only one which gives $(l_0 = -1)$ is

$$(r = 0, l = 0) \quad (2.20)$$

The $(r = 0, l = -1)$ case is ruled out by twist symmetry [1], therefore, $\Phi^{(2)}$ can be written as

$$\begin{aligned} \Phi^{(2)} = & - \sum_{r=0}^{\infty} \frac{1}{(-2)^r r!} (L^+)^r \sum_{l=-\infty}^1 \sum_{m=-\infty}^{-1} \tilde{c}_l \tilde{c}_1 \tilde{V}_m \tilde{V}_{-1} |0\rangle \\ & - \sum_{r'} \frac{1}{(-2)^{r'} r'!} (L^+)^{r'} \sum_{l'} \tilde{c}_{l'} \tilde{c}_1 |0\rangle \\ & + \left(-\tilde{c}_0 \tilde{c}_1 |0\rangle + \frac{1}{2} L^+ \tilde{c}_0 \tilde{c}_1 |0\rangle \right) \end{aligned} \quad (2.21)$$

where the primed indices are the corresponding unprimed indices without the cases which give $(l_0 = 0)$ or $(l_0 = -1)$. The last line is BRST exact so that we can write

$$\begin{aligned} \Phi^{(2)} = & - \sum_{r=0}^{\infty} \frac{1}{(-2)^r r!} (L^+)^r \sum_{l=-\infty}^1 \sum_{m=-\infty}^{-1} \tilde{c}_l \tilde{c}_1 \tilde{V}_m \tilde{V}_{-1} |0\rangle \\ & - \sum_{r'} \frac{1}{(-2)^{r'} r'!} (L^+)^{r'} \sum_{l'} \tilde{c}_{l'} \tilde{c}_1 |0\rangle \\ & + Q_B \left(\tilde{c}_1 |0\rangle - \frac{1}{2} L^+ \tilde{c}_1 |0\rangle \right) \\ = & Q_B \left(\tilde{c}_1 |0\rangle - \frac{1}{2} L^+ \tilde{c}_1 |0\rangle \right) + \Phi_{>}^{(2)} \end{aligned} \quad (2.22)$$

where $\Phi_{>}^{(2)}$ contains only $l_0 > 0$ states. Up to some Q_B closed term $\Psi^{(2)}$ is given by

$$\Psi^{(2)} = \tilde{c}_1|0\rangle - \frac{1}{2}L^+\tilde{c}_1|0\rangle + \Psi_{>}^{(2)} \quad (2.23)$$

where $\Psi_{>}^{(2)}$ satisfies $Q_B\Psi_{>}^{(2)} = \Phi_{>}^{(2)}$. Assuming $\Psi_{>}^{(2)}$ is in the Schnabl gauge we can write

$$\begin{aligned} \Psi^{(2)} &= \tilde{c}_1|0\rangle - \frac{1}{2}L^+\tilde{c}_1|0\rangle + \int_0^\infty dT B e^{-TL} \Phi_{>}^{(2)} \\ &= \tilde{c}_1|0\rangle - \frac{1}{2}L^+\tilde{c}_1|0\rangle + \lim_{\Lambda \rightarrow \infty} \int_0^\Lambda dT B e^{-TL} [\Phi^{(2)} + \left(\tilde{c}_0\tilde{c}_1|0\rangle - \frac{1}{2}L^+\tilde{c}_0\tilde{c}_1|0\rangle \right)] \end{aligned}$$

Replacing $\int_0^\Lambda dT B e^{-TL}$ by $\frac{B}{L} - e^{-\Lambda L} \frac{B}{L}$ on the terms with $l_0 = 0$ and $l_0 = -1$ we obtain

$$\begin{aligned} \Psi^{(2)} &= \lim_{\Lambda \rightarrow \infty} \left(\int_0^\Lambda dT B e^{-TL} \Phi^{(2)} + e^\Lambda \tilde{c}_1|0\rangle - \frac{1}{2}\Lambda [L^+\tilde{c}_1|0\rangle + B^+\tilde{c}_0\tilde{c}_1|0\rangle] - \frac{1}{2}L^+\tilde{c}_1|0\rangle \right) \\ &= \lim_{\Lambda \rightarrow \infty} \left(-\frac{1}{2}(\Lambda+1)L^+\tilde{c}_1|0\rangle - \frac{1}{2}\Lambda B^+\tilde{c}_0\tilde{c}_1|0\rangle + e^\Lambda \tilde{c}_1|0\rangle - \int_{e^{-\Lambda}}^1 dt \Psi^{(1)} * U_t^\dagger U_t|0\rangle * B_L^+ \Psi^{(1)} \right) \end{aligned} \quad (2.24)$$

This solution is obtained by inverting L only on the positive eigenvalue terms of $\Phi^{(2)}$ so that it is regular. We also notice that one can not fit the entire $\Psi^{(2)}$ into the Schnabl gauge, only the portion which is related to the positive eigenvalue terms of $\Phi^{(2)}$ can satisfy the gauge condition. The fact that this solution is regular will be apparent when we will use it to calculate the tachyon profile of the array of D24-brane solutions in the next section.

Now we use the identity (see [1])

$$\phi_1 * B_L^+ \phi_2 = (-1)^{\phi_1} B_L^+ (\phi_1 * \phi_2) - (-1)^{\phi_1} (B_1 \phi_1) * \phi_2 \quad (2.25)$$

and the fact that $B_1 U_t^\dagger U_t|0\rangle = 0$ and write $\Psi^{(2)}$ as

$$\begin{aligned} \Psi^{(2)} &= \lim_{\Lambda \rightarrow \infty} \left(\int_{e^{-\Lambda}}^1 dt \{ B_L^+ [\Psi^{(1)} * U_t^\dagger U_t|0\rangle * \Psi^{(1)}] - [B_1 \Psi^{(1)}] * U_t^\dagger U_t|0\rangle * \Psi^{(1)} \} \right. \\ &\quad \left. + e^\Lambda \tilde{c}_1|0\rangle - \frac{1}{2}(\Lambda+1)L^+\tilde{c}_1|0\rangle - \frac{1}{2}\Lambda B^+\tilde{c}_0\tilde{c}_1|0\rangle \right). \end{aligned} \quad (2.26)$$

This last form is convenient to calculate $\Phi^{(3)}$ which is given by

$$\Phi^{(3)} = -\Psi^{(1)} * \Psi^{(2)} - \Psi^{(2)} * \Psi^{(1)}. \quad (2.27)$$

With the help of the identity (2.25) again, we obtain

$$\begin{aligned} \Phi^{(3)} &= \lim_{\Lambda \rightarrow \infty} \left(\int_{e^{-\Lambda}}^1 dt \{ B_L^+ [\Psi^{(1)} * \Psi^{(1)} * U_t^\dagger U_t|0\rangle * \Psi^{(1)}] - [B_1 \Psi^{(1)}] * \Psi^{(1)} * U_t^\dagger U_t|0\rangle * \Psi^{(1)} \right. \\ &\quad \left. + \Psi^{(1)} * [B_1 \Psi^{(1)}] * U_t^\dagger U_t|0\rangle * \Psi^{(1)} \} - e^\Lambda \Psi^{(1)} * \tilde{c}_1|0\rangle + \frac{1}{2}\Psi^{(1)} * L^+\tilde{c}_1|0\rangle \right. \\ &\quad \left. - \frac{1}{2}\Lambda Q_B [\Psi^{(1)} * B^+\tilde{c}_1|0\rangle] \right) \\ &= \lim_{\Lambda \rightarrow \infty} \left(\int_{e^{-\Lambda}}^1 dt \{ B_L^+ [\Psi^{(1)} * U_t^\dagger U_t|0\rangle * \Psi^{(1)} * \Psi^{(1)}] - [B_1 \Psi^{(1)}] * U_t^\dagger U_t|0\rangle * \Psi^{(1)} * \Psi^{(1)} \} \right. \\ &\quad \left. + e^\Lambda \tilde{c}_1|0\rangle * \Psi^{(1)} - \frac{1}{2}L^+\tilde{c}_1|0\rangle * \Psi^{(1)} - \frac{1}{2}\Lambda Q_B [B^+\tilde{c}_1|0\rangle * \Psi^{(1)}] \right). \end{aligned} \quad (2.28)$$

Now we will need the following re-writings

$$\begin{aligned} L^+ \phi_1 * \phi_2 &= -2 \frac{\partial}{\partial s} U_s^\dagger U_s \phi_1 * \phi_2 \big|_{s=2} \\ \phi_1 * L^+ \phi_2 &= -2 \frac{\partial}{\partial s} \phi_1 * U_s^\dagger U_s \phi_2 \big|_{s=2} . \end{aligned} \quad (2.29)$$

Therefore,

$$\begin{aligned} \Phi^{(3)} &= \lim_{\Lambda \rightarrow \infty} \left(\int_{e^{-\Lambda}}^1 dt \left\{ B_L^+ [\Psi^{(1)} * \Psi^{(1)} * U_t^\dagger U_t |0\rangle * \Psi^{(1)}] - [B_1 \Psi^{(1)}] * \Psi^{(1)} * U_t^\dagger U_t |0\rangle * \Psi^{(1)} \right. \right. \\ &\quad + \Psi^{(1)} * [B_1 \Psi^{(1)}] * U_t^\dagger U_t |0\rangle * \Psi^{(1)} \left. \right\} - e^\Lambda \Psi^{(1)} * \tilde{c}_1 |0\rangle - \frac{\partial}{\partial s} [\Psi^{(1)} * U_s^\dagger U_s \tilde{c}_1 |0\rangle] \big|_{s=2} \\ &\quad - \frac{1}{2} \Lambda Q_B [\Psi^{(1)} * B^+ \tilde{c}_1 |0\rangle] \left. \right) \\ &\quad - \lim_{\Lambda \rightarrow \infty} \left(\int_{e^{-\Lambda}}^1 dt \left\{ B_L^+ [\Psi^{(1)} * U_t^\dagger U_t |0\rangle * \Psi^{(1)} * \Psi^{(1)}] - [B_1 \Psi^{(1)}] * U_t^\dagger U_t |0\rangle * \Psi^{(1)} * \Psi^{(1)} \right\} \right. \\ &\quad + e^\Lambda \tilde{c}_1 |0\rangle * \Psi^{(1)} + \frac{\partial}{\partial s} [U_s^\dagger U_s \tilde{c}_1 |0\rangle * \Psi^{(1)}] \big|_{s=2} - \frac{1}{2} \Lambda Q_B [B^+ \tilde{c}_1 |0\rangle * \Psi^{(1)}] \left. \right) \end{aligned} \quad (2.30)$$

Since $B_1 \Psi^{(1)} = V(0)|0\rangle$ we can use the standard formula for star product of wedge states with insertions to perform the star product. As usual, our aim is to single out the terms with negative or zero eigenvalues of L so that we can use the Schwinger representation (2.5) of Q_B on the remaining terms of $\Phi^{(3)}$ to obtain $\Psi^{(3)}$. It can be easily verified that the Q_B exact terms in $\Phi^{(3)}$ do not contain $l_0 \leq 0$ term, therefore, we will leave these terms as they are.

$$\begin{aligned} \Phi^{(3)} &= \lim_{\Lambda \rightarrow \infty} \left[\int_{e^{-\Lambda}}^1 dt \left\{ B_L^+ U_{t+3}^\dagger U_{t+3} \tilde{c} \tilde{V}(t+1) \tilde{c} \tilde{V}(t) \tilde{c} \tilde{V}(0) |0\rangle \right. \right. \\ &\quad - U_{t+3}^\dagger U_{t+3} \tilde{V}(t+1) \tilde{c} \tilde{V}(t) \tilde{c} \tilde{V}(0) |0\rangle \\ &\quad + U_{t+3}^\dagger U_{t+3} \tilde{c} \tilde{V}(t+1) \tilde{V}(t) \tilde{c} \tilde{V}(0) |0\rangle \left. \right\} \\ &\quad - e^\Lambda U_3^\dagger U_3 \tilde{c} \tilde{V}(1) \tilde{c}(0) |0\rangle + \frac{1}{2} U_3^\dagger U_3 L^+ \tilde{c} \tilde{V}(1) \tilde{c}(0) |0\rangle \\ &\quad - \frac{1}{2} U_3^\dagger U_3 \partial(\tilde{c} \tilde{V})(1) \tilde{c}(0) |0\rangle - \frac{1}{2} \Lambda Q_B [\Psi^{(1)} * B^+ \tilde{c}_1 |0\rangle] \left. \right] \\ &\quad - \lim_{\Lambda \rightarrow \infty} \left[\int_{e^{-\Lambda}}^1 dt \left\{ B_L^+ U_{t+3}^\dagger U_{t+3} \tilde{c} \tilde{V}(t+1) \tilde{c} \tilde{V}(1) \tilde{c} \tilde{V}(0) |0\rangle \right. \right. \\ &\quad - U_{t+3}^\dagger U_{t+3} \tilde{V}(t+1) \tilde{c} \tilde{V}(1) \tilde{c} \tilde{V}(0) |0\rangle \left. \right\} \\ &\quad + e^\Lambda U_3^\dagger U_3 \tilde{c}(1) \tilde{c} \tilde{V}(0) |0\rangle - \frac{1}{2} U_3^\dagger U_3 L^+ \tilde{c}(1) \tilde{c} \tilde{V}(0) |0\rangle \\ &\quad + \frac{1}{2} U_3^\dagger U_3 \partial \tilde{c}(1) \tilde{c} \tilde{V}(0) |0\rangle - \frac{1}{2} \Lambda Q_B [B^+ \tilde{c}_1 |0\rangle * \Psi^{(1)}] \left. \right] \end{aligned} \quad (2.31)$$

As we did for $\Phi^{(2)}$, after expanding both \tilde{c} and \tilde{V} in modes and also expanding $U_s^\dagger U_s$ in power of L^+ we see that each term in the multiple summation is an eigenstate of L . We

would like to focus on the terms which contain $l_0 \leq 0$ and which are not Q_B exact. Here we see that the e^Λ , the $(\partial c)V$ and ∂c terms contain such states. It is also easy to see that some contribution comes from the lines 2, 3 and 7. Using the commutation relation for the V modes we can separate these terms from the others so that

$$\begin{aligned} \Phi^{(3)} = & \lim_{\Lambda \rightarrow \infty} \left(\left[1 - 2e^\Lambda + \int_{e^{-\Lambda}}^1 dt \ f(t) \right] \tilde{c}_0 \tilde{c}_1 \tilde{V}_{-1} |0\rangle + \Phi_{>}^{(3)}(\text{non-exact}) \right. \\ & \left. - \frac{1}{2} \Lambda Q_B [\Psi^{(1)} * B^+ \tilde{c}_1 |0\rangle] - B^+ \tilde{c}_1 |0\rangle * \Psi^{(1)} \right) \end{aligned} \quad (2.32)$$

where $\Phi_{>}^{(3)}(\text{non-exact})$ contains only terms with $l_0 > 0$ and are not Q_B exact and

$$f(t) = 2 + \frac{2}{t^2} + \frac{2}{(1+t)^2}. \quad (2.33)$$

Here we notice that unlike the $\Psi^{(2)}$ case, now we have Q_B non-exact $l_0 = 0$ terms, therefore, we can not tell apart every term with $l_0 = 0$ of $\Psi^{(3)}$. However, still there is a piece of (2.28) which is Q_B exact. It is convenient to write $\Phi^{(3)}$ as

$$\Phi^{(3)} = - \lim_{\Lambda \rightarrow \infty} \frac{1}{2} \Lambda Q_B \left(\Psi^{(1)} * B^+ \tilde{c}_1 |0\rangle - B^+ \tilde{c}_1 |0\rangle * \Psi^{(1)} \right) + \Phi_{rest}^{(3)}. \quad (2.34)$$

With this we can see that the most general $\Psi^{(3)}$, up to some Q_B closed addition, is

$$\Psi^{(3)} = - \lim_{\Lambda \rightarrow \infty} \frac{1}{2} \Lambda \left(\Psi^{(1)} * B^+ \tilde{c}_1 |0\rangle - B^+ \tilde{c}_1 |0\rangle * \Psi^{(1)} \right) + \Psi_{rest}^{(3)}, \quad (2.35)$$

where $\Psi_{rest}^{(3)}$ is defined as

$$Q_B \Psi_{rest}^{(3)} = \Phi_{rest}^{(3)}. \quad (2.36)$$

We assume that $\Psi_{rest}^{(3)}$ is in the Schnabl gauge, so we can formally put $\Psi^{(3)}$ as

$$\begin{aligned} \Psi_0^{(3)} = & - \lim_{\Lambda \rightarrow \infty} \frac{1}{2} \Lambda \left(\Psi^{(1)} * B^+ \tilde{c}_1 |0\rangle - B^+ \tilde{c}_1 |0\rangle * \Psi^{(1)} \right) \\ & + \lim_{\Gamma \rightarrow \infty} \left(\int_0^\Gamma dT \ B e^{-TL} \Phi_{rest}^{(3)} \right) \end{aligned} \quad (2.37)$$

This has Q_B closed divergent term which arise from some of the $l_0 = 0$ terms of $\Phi_{rest}^{(3)}$ and needs to be regularized. From (2.32) it is not difficult to realize that the regularized $\Psi^{(3)}$ will be

$$\begin{aligned} \Psi_{reg}^{(3)} = & - \lim_{\Lambda \rightarrow \infty} \frac{1}{2} \Lambda \left(\Psi^{(1)} * B^+ \tilde{c}_1 |0\rangle - B^+ \tilde{c}_1 |0\rangle * \Psi^{(1)} \right) \\ & + \lim_{\Gamma \rightarrow \infty} \left(\int_0^\Gamma dT \ B e^{-TL} \Phi_{rest}^{(3)} - \lim_{\Lambda \rightarrow \infty} \left[-2e^\Lambda + \int_{e^{-\Lambda}}^1 dt \ f(t) \right] \Gamma \tilde{c}_1 \tilde{V}_{-1} |0\rangle \right) \end{aligned} \quad (2.38)$$

Note that the added counter terms are all Q_B closed and are also in the Schnabl gauge, therefore, they will not affect the equation of motion as well as the gauge condition. From equations (2.28) and (2.34) we can easily read $\Phi_{rest}^{(3)}$ and we finally obtain

$$\begin{aligned}
\Psi_{reg}^{(3)} = & - \lim_{\Lambda \rightarrow \infty} \frac{1}{2} \Lambda \left(\Psi^{(1)} * B^+ \tilde{c}_1 |0\rangle - B^+ \tilde{c}_1 |0\rangle * \Psi^{(1)} \right) \\
& + \lim_{\Gamma \rightarrow \infty} \lim_{\Lambda \rightarrow \infty} \left\{ - \left[-2e^\Lambda + \int_{e^{-\Lambda}}^1 dt \ f(t) \right] \Gamma \tilde{c}_1 \tilde{V}_{-1} |0\rangle \right. \\
& - e^\Lambda \int_{e^{-\Gamma}}^1 dt_2 \ \frac{1}{t_2} \left[\Psi^{(1)} * U_{t_2}^\dagger U_{t_2} |0\rangle * B_L^+ \tilde{c}_1 |0\rangle + \tilde{c}_1 |0\rangle * U_{t_2}^\dagger U_{t_2} |0\rangle * B_L^+ \Psi^{(1)} \right] \\
& + \frac{1}{2} \int_{e^{-\Gamma}}^1 dt_2 \left(\frac{1}{t_2} \left[-\Psi^{(1)} * U_{t_2}^\dagger U_{t_2} |0\rangle * B^+ \tilde{c}_1 |0\rangle + B^+ \tilde{c}_1 |0\rangle * U_{t_2}^\dagger U_{t_2} |0\rangle * \Psi^{(1)} \right] \right. \\
& + \left. \Psi^{(1)} * U_{t_2}^\dagger U_{t_2} |0\rangle * B_L^+ L^+ \tilde{c}_1 |0\rangle + L^+ \tilde{c}_1 |0\rangle * U_{t_2}^\dagger U_{t_2} |0\rangle * B_L^+ \Psi^{(1)} \right) \\
& + \int_{e^{-\Lambda}}^1 dt_1 \int_{e^{-\Gamma}}^1 dt_2 \ t_2 \left[\Psi^{(1)} * U_{t_2}^\dagger U_{t_2} |0\rangle * (-B_L^+) \Psi^{(1)} * U_{t_1 t_2}^\dagger U_{t_1 t_2} |0\rangle * (-B_L^+) \Psi^{(1)} \right] \\
& + \left. \int_{e^{-\Lambda}}^1 dt_1 \int_{e^{-\Gamma}}^1 dt_2 \ t_2 \left[\Psi^{(1)} * U_{t_1 t_2}^\dagger U_{t_1 t_2} |0\rangle * (-B_L^+) \Psi^{(1)} * U_{t_2}^\dagger U_{t_2} |0\rangle * (-B_L^+) \Psi^{(1)} \right] \right\} \\
& \tag{2.39}
\end{aligned}$$

In the third and the fourth lines it is clear that the integrands are not well defined in the region $t_2 \rightarrow 0$. However, the singularities coming from this region are cancelled partly by the corresponding counter terms in the second line, the $(-2e^\Lambda)$ term and partly by similar divergences coming from the last two lines. There are other divergences arising from the last two lines. These will be cancelled by the remaining part of the counter term in the second line and there is no more divergence related to $t_2 \rightarrow 0$. Since the divergences related to $t_1 \rightarrow 0$ has already been regularized at level 2, this result is perfectly regular. We will demonstrate this cancellation of divergences in the next section using a particular example. We would like to emphasize also, like it is at level 2 here again the entire solution can not be in the Schnabl gauge, only the part which is obtained from the Q_B non-exact piece of $\Phi^{(3)}$ is in the gauge.

For level 4 calculation we would like to focus entirely on the terms which have zero or negative L eigenvalues. Separating these terms from the rest we can write $\Phi^{(4)}$ as

$$\begin{aligned}
\Phi^{(4)} = & - \left[\Psi^{(3)} * \Psi^{(1)} + \Psi^{(1)} * \Psi^{(3)} + \Psi^{(2)} * \Psi^{(2)} \right]_{>} \\
& + \lim_{\Lambda \rightarrow \infty} \lim_{\Gamma \rightarrow \infty} \left\{ \left[-\Lambda + \left(4e^\Lambda - 2 \int_{e^{-\Lambda}}^1 dt f(t) \right) \Gamma + e^\Lambda \int_{e^{-\Gamma}}^1 dt_2 \left(-\frac{2}{t_2} - \frac{2}{t_2(1+t_2)^2} \right) \right. \right. \\
& + \int_{e^{-\Gamma}}^1 dt_2 \left(-\frac{1}{t_2} - \frac{1-t_2}{t_2(1+t_2)^2} \right) - \int_{e^{-\Gamma}}^1 dt_2 \frac{t_2-3}{(1+t_2)^3} \\
& + \int_{e^{-\Lambda}}^1 dt_1 \int_{e^{-\Gamma}}^1 dt_2 t_2 \left(\frac{2}{t_2^2(l_2+1)^2} + \frac{2}{l_2^2(t_2+1)^2} + \frac{2}{(l_2-t_2)^2} + t_2 \rightarrow t_1 t_2 \right) \\
& - \int_{e^{-\Lambda}}^1 dt_1 \int_{e^{-\Lambda}}^1 dt_2 \left(\frac{t_2}{(l_2'+1)^2} + \frac{t_2}{(t_1+1)^2(t_2+1)^2} + \frac{1}{t_1 t_2^2} \right) \\
& + \int_{e^{-\Lambda}}^1 dt_1 \int_{e^{-\Lambda}}^1 dt_2 \left(\frac{t_2+1}{(l_2'+1)^2} + \frac{t_2+1}{t_1^2 t_2^2} + \frac{1}{(t_2+1)(t_1+1)^2} \right) \\
& - e^\Lambda \left(- \int_{e^{-\Lambda}}^1 dt_1 \frac{1}{t_1^2} + \frac{e^{-\Lambda}}{2} \int_{e^{-\Lambda}}^1 dt_1 \frac{1}{t_1^2} - \int_{e^{-\Lambda}}^1 dt_2 \frac{1}{t_2} + \int_{e^{-\Lambda}}^1 dt_2 \frac{t_2+1}{t_2^2} - e^\Lambda - (\Lambda+1) \right) \\
& - \frac{(\Lambda+1)}{2} \int_{e^{-\Lambda}}^1 dt_2 \frac{1}{t_2} - \frac{\Lambda}{2} \int_{e^{-\Lambda}}^1 dt_2 \frac{1}{t_2^2} + \frac{\Lambda(\Lambda+1)}{4} \left. \right] \mathbf{Q}_B \mathbf{L}^+ \tilde{\mathbf{c}}_1 |0\rangle \\
& + \left[\Lambda + \left(-2e^\Lambda + \int_{e^{-\Lambda}}^1 dt f(t) \right) \Gamma + e^\Lambda \int_{e^{-\Gamma}}^1 dt_2 \left(\frac{1+t_2}{t_2} + \frac{1}{t_2(1+t_2)} \right) \right. \\
& + \int_{e^{-\Gamma}}^1 dt_2 \left(\frac{1+t_2}{2t_2} + \frac{1-t_2}{2t_2(1+t_2)} \right) + \int_{e^{-\Gamma}}^1 dt_2 \left(1 + \frac{t_2-1}{2(1+t_2)} \right) \\
& - \int_{e^{-\Lambda}}^1 dt_1 \int_{e^{-\Gamma}}^1 dt_2 t_2(l_2+1) \left(\frac{1}{t_2^2(l_2+1)^2} + \frac{1}{l_2^2(t_2+1)^2} + \frac{1}{(l_2-t_2)^2} + t_2 \rightarrow t_1 t_2 \right) \\
& + \int_{e^{-\Lambda}}^1 dt_1 \int_{e^{-\Lambda}}^1 dt_2 \frac{l_2'+1}{2} \left(\frac{t_2}{(l_2'+1)^2} + \frac{t_2}{(t_1+1)^2(t_2+1)^2} + \frac{1}{t_1 t_2^2} \right) \\
& - \int_{e^{-\Lambda}}^1 dt_1 \int_{e^{-\Lambda}}^1 dt_2 \frac{l_2'+1}{2} \left(\frac{t_2+1}{(l_2'+1)^2} + \frac{t_2+1}{t_1^2 t_2^2} + \frac{1}{(t_2+1)(t_1+1)^2} \right) \\
& - e^\Lambda \int_{e^{-\Lambda}}^1 dt_1 \frac{t_1+1}{2t_1^2} + \frac{\Lambda+1}{2} \int_{e^{-\Lambda}}^1 dt_1 \frac{t_1+3}{2t_1^2} - \frac{\Lambda}{2} \int_{e^{-\Lambda}}^1 dt_1 \frac{t_1+1}{2t_1^2} \\
& - e^\Lambda \left(\int_{e^{-\Lambda}}^1 dt_2 \left(\frac{t_2+1}{2t_2} - \frac{(t_2+1)^2}{2t_2^2} \right) + \frac{e^\Lambda}{2} + \frac{3(\Lambda+1)}{2} + e^{-\Lambda} \frac{(\Lambda+1)}{2} \int_{e^{-\Lambda}}^1 dt_2 \frac{1}{t_2} \right) \\
& + \frac{3(\Lambda+1)}{4} \int_{e^{-\Lambda}}^1 dt_2 \frac{t_2+1}{t_2} - (\Lambda+1)^2 + \frac{\Lambda}{2} \int_{e^{-\Lambda}}^1 dt_2 \frac{t_2+1}{2t_2^2} \\
& \left. - \frac{3\Lambda(\Lambda+1)}{8} \right] \mathbf{Q}_B \mathbf{L}^+ \tilde{\mathbf{c}}_1 |0\rangle \Big\} \tag{2.40}
\end{aligned}$$

where $l_2 = t_2 + t_1 t_2$ and $l_2' = t_1 + t_2$. Since the terms in the square brackets are just numerical factors we may write $\Phi^{(4)}$ as

$$\Phi^{(4)} = \Phi_{>}^{(4)} + A \mathbf{Q}_B \tilde{\mathbf{c}}_1 |0\rangle + B \mathbf{Q}_B \mathbf{L}^+ \tilde{\mathbf{c}}_1 |0\rangle. \tag{2.41}$$

We see that $\Phi^{(4)}$ has the same form as $\Phi^{(2)}$ such that the terms with zero or negative L eigenvalues are Q_B exact. Therefore, we follow the same procedure as in level two to solve for $\Psi^{(4)}$. Up to some Q_B closed term the ansatz for $\Psi^{(4)}$ is

$$\Psi^{(4)} = A\tilde{c}_1|0\rangle + BL^+\tilde{c}_1|0\rangle + \Psi_{>}^{(4)} \quad (2.42)$$

where $\Psi_{>}^{(4)}$ satisfies $Q_B\Psi_{>}^{(4)} = \Phi_{>}^{(4)}$. Assuming $\Psi_{>}^{(4)}$ is in the Schnabl gauge we can write

$$\Psi^{(4)} = A\tilde{c}_1|0\rangle + BL^+\tilde{c}_1|0\rangle + \int_0^\infty dT B e^{-TL} \Phi_{>}^{(4)} \quad (2.43)$$

In obtaining this solution we applied L^{-1} or its Schwinger form only on terms with positive definite L eigenvalues. Therefore, at this level we didn't produce any new divergent term. Since we have already regularized all the lower level $\Psi^{(i)}$'s it is clear that $\Psi^{(4)}$ is regular. Like the lower levels we see that also at this level the solution contains a gauge condition violating term, which is a characteristic of the solutions with singular OPE.

Now let's generalize our procedure to arbitrary level. By now it is clear that divergences arise only when there are zero or negative L eigenvalue terms in $\Phi^{(n)}$. Noting that $\Phi^{(n)}$ has to be of ghost number two and has to be twist even, one can easily see that the only terms which can appear in $\Phi^{(n)}$ and can have a negative or zero eigenvalue are $(\tilde{c}_0\tilde{c}_1|0\rangle, L^+\tilde{c}_0\tilde{c}_1|0\rangle, \tilde{c}_0\tilde{c}_1\tilde{V}_{-1}|0\rangle)$, which are exactly what we have at levels two and three. In particular, if n is even only the first two of these terms (which are Q_B exact) appear. The reason is that applying the commutation relation for each pair of V kills all the V operators and hence the term of the third kind can not appear. In this case we follow the procedure of level two to solve for $\Psi^{(n)}$. For odd n , where we have odd number of V operators and the pairing will leave one V , only the last term appears and it gives rise to a new divergent term in $\Psi^{(n)}$. However, this new divergent term is Q_B closed as well as satisfies the Schnabl gauge so that we can subtract it out to get a regular solution. Therefore, our procedure can be used at any level.

3. The tachyon profile

In this section we consider the special case of a marginal deformation corresponding to a periodic array of D24-branes. The dimension one boundary matter primary operator $V(z)$ giving such a solution is

$$V(z) = \frac{1}{\sqrt{2}}[V^+(z) + V^-(z)], \quad \text{with } V^\pm = e^{\pm iX(z)} \quad (3.1)$$

where we choose $\alpha' = 1$ and $X(z) = X^{25}(z)$. We can easily see that the OPE of V with itself is given by (2.15) and hence it is an example of singular OPE solutions we saw in section 2. Our aim in this section is to calculate the x dependence of the tachyon field level by level and verify that the solutions are regular and they indeed correspond to arrays of D24-branes. The calculation beyond the third level is too complicated so we restrict our treatment in this section to the first three levels. Actually, the overall shape of the tachyon profile does not change when we consider higher level contributions, what changes

is the depth of its minima, to which we are not intending to associate any physical meaning for the reason we will give in the discussion section.

Since the result at level one is trivial we start with level two calculations. At level two x dependence of $\Psi^{(2)}$ must be of the form

$$\Psi^{(2)} = \left(e^{2iX(0)} + e^{-2iX(0)} \right) \left[\beta_2^2 c_1 |0\rangle + \dots \right] + \left[\beta_0^2 c_1 |0\rangle + \dots \right]. \quad (3.2)$$

The dots indicate higher level space-time fields and the coefficients β_n^2 are given by

$$\beta_n^2 = \langle \phi_{\pm n}, \Psi^{(2)} \rangle, \quad \phi_{\pm n} = e^{\pm inX(0)} c \partial c(0) |0\rangle \quad (3.3)$$

where we have ignored the irrelevant space time volume factor. By momentum conservation β_2^2 gets a contribution only from the last term of (2.24) which is given by

$$\begin{aligned} \beta_2^2 &= \frac{1}{2} \left\langle \phi_{-2}, \lim_{\Lambda \rightarrow \infty} \int_{e^{-\Lambda}}^1 dt \ c V^+(0) |0\rangle * U_t^\dagger U_t |0\rangle * (-B_L^+) c V^+(0) |0\rangle \right\rangle \\ &= \frac{1}{2} \left\langle \phi_2, \lim_{\Lambda \rightarrow \infty} \int_{e^{-\Lambda}}^1 dt \ c V^-(0) |0\rangle * U_t^\dagger U_t |0\rangle * (-B_L^+) c V^-(0) |0\rangle \right\rangle \end{aligned}$$

Each of the V^\pm 's gives the regular OPE solutions and the above result have been calculated in [15], and the answer is

$$\beta_2^2 = \frac{1}{2}(0.15206). \quad (3.4)$$

β_0^2 gets a contribution from all the terms in (2.24). Using the definitions

$$L^+ = -2(K_1^L - K_1), \quad B^+ = -2(B_1^L - B_1) \quad (3.5)$$

and noting that $(K_1 c_1 |0\rangle + B_1 c_0 c_1 |0\rangle = 0)$ we can rewrite $\Psi^{(2)}$ in the following more convenient way:

$$\begin{aligned} \Psi^{(2)} &= \lim_{\Lambda \rightarrow \infty} \left(\Lambda \psi'_0 - \frac{1}{2} L^+ \tilde{c}_1 |0\rangle + e^\Lambda \tilde{c}_1 |0\rangle - \int_{e^{-\Lambda}}^1 dt \ \Psi^{(1)} * U_t^\dagger U_t |0\rangle * B_L^+ \Psi^{(1)} \right) \\ &= \lim_{\Lambda \rightarrow \infty} \left(-\Lambda \psi'_0 + e^\Lambda \exp \left[-\frac{e^{-\Lambda} L^+}{2} \right] \tilde{c}_1 |0\rangle - \int_{e^{-\Lambda}}^1 dt \ \Psi^{(1)} * U_t^\dagger U_t |0\rangle * B_L^+ \Psi^{(1)} \right) \end{aligned}$$

where $(\psi'_0 = K_1^L c_1 |0\rangle + B_1^L c_0 c_1 |0\rangle)$ is defined in [2]. With the help of the identity $(L^+ = 2L_L^+ + K_1)$, in the limit $\Lambda \rightarrow \infty$ it is not difficult to show that

$$\exp \left[-\frac{e^{-\Lambda} L^+}{2} \right] \tilde{c}_1 |0\rangle = U_{e^{-\Lambda}+1}^\dagger U_{e^{-\Lambda}+1} |0\rangle * \left(\tilde{c}_1 |0\rangle - \frac{1}{2} e^{-\Lambda} \tilde{c}_0 |0\rangle \right) \quad (3.6)$$

so that

$$\begin{aligned} \Psi^{(2)} &= \lim_{\Lambda \rightarrow \infty} \left[-\Lambda \psi'_0 + e^\Lambda U_{e^{-\Lambda}+1}^\dagger U_{e^{-\Lambda}+1} |0\rangle * \left(\tilde{c}_1 |0\rangle - \frac{1}{2} e^{-\Lambda} \tilde{c}_0 |0\rangle \right) \right. \\ &\quad \left. + \int_{e^{-\Lambda}}^1 dt \ \Psi^{(1)} * U_t^\dagger U_t |0\rangle * (-B_L^+) \Psi^{(1)} \right]. \end{aligned} \quad (3.7)$$

Therefore,

$$\begin{aligned} \beta_0^2 = & \lim_{\Lambda \rightarrow \infty} \left(-\Lambda \langle \phi_0, \psi'_0 \rangle - e^\Lambda \left\langle f \circ \phi_0(0) \left(\tilde{c} - \frac{1}{2} e^{-\Lambda} \partial \tilde{c} \right) (e^{-\Lambda} + 1) \right\rangle_{e^{-\Lambda}+2} \right. \\ & \left. + \int_{e^{-\Lambda}}^1 dt \left\langle f \circ \phi_0(0) \tilde{c} \tilde{V}^+(1) \mathcal{B} \tilde{c} \tilde{V}^-(t+1) \right\rangle_{t+2} \right). \end{aligned} \quad (3.8)$$

The subscripts indicate the width of the strip over which the correlators are taken. Noting that $\phi_0 = Q_B c(0)|0\rangle$ we have

$$\langle \phi_0, \psi'_0 \rangle = \langle c_{-1}, Q_B \psi'_0 \rangle = 0 \quad (3.9)$$

After a simple calculation the remaining terms in the first line of (3.8) gives

$$e^\Lambda \left\langle f \circ \phi_0(0) \left(\tilde{c} - \frac{1}{2} e^{-\Lambda} \partial \tilde{c} \right) (e^{-\Lambda} + 1) \right\rangle_{e^{-\Lambda}+2} = \frac{2}{\pi} (e^\Lambda + 1) + \mathcal{O}(e^{-\Lambda}). \quad (3.10)$$

Note that when we do the star product of wedge states with insertions (eq. 2.11) we insert the operator of the last state in the star product, first on the strip obtained by gluing together the strips of the individual state. This operator ordering is opposite to the one we use when we calculate the correlator in (3.8) and as a result we got an extra minus sign. The ghost part of the last line of (3.8) have been calculated in [15] and the matter part calculation is straight forward. Finally, we obtain

$$\begin{aligned} \int_{e^{-\Lambda}}^1 dt \left\langle f \circ \phi_0(0) \tilde{c} \tilde{V}^+(1) \mathcal{B} \tilde{c} \tilde{V}^-(t+1) \right\rangle_{t+2} = & \int_{e^{-\Lambda}}^1 dt \frac{\pi}{t+2} \left[1 - \frac{2+t}{2\pi} \sin \left(\frac{2\pi}{2+t} \right) \right] \\ & \times \sin^2 \left(\frac{\pi}{2+t} \right) \sin^{-2} \left(\frac{\pi t}{2+t} \right) \end{aligned} \quad (3.11)$$

Putting every thing together, and using Mathematica we obtain

$$\beta_0^2 = -\frac{\sqrt{27}}{4} = -1.29904 \quad (3.12)$$

which is regular as we anticipated.

To level 2 the tachyon profile is given by

$$T(x) = -\cos(x) + (0.15206)\cos(2x) - 1.29904 \quad (3.13)$$

if we choose $\lambda = -1$ and

$$T(x) = \cos(x) + (0.15206)\cos(2x) - 1.29904 \quad (3.14)$$

if we choose $\lambda = +1$.

Now lets proceed to level three calculations which should be of the form

$$\Psi^{(3)} = \left(e^{3iX(0)} + e^{-3iX(0)} \right) [\beta_3^3 c_1 |0\rangle + \dots] + \left(e^{iX(0)} + e^{-iX(0)} \right) [\beta_1^3 c_1 |0\rangle + \dots] \quad (3.15)$$

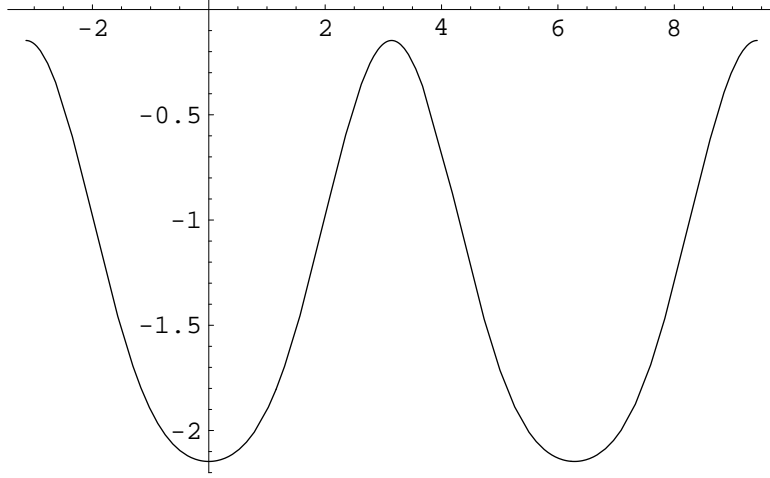


Figure 1: *The level 2 approximation of the tachyon profile for $\lambda = -1$*

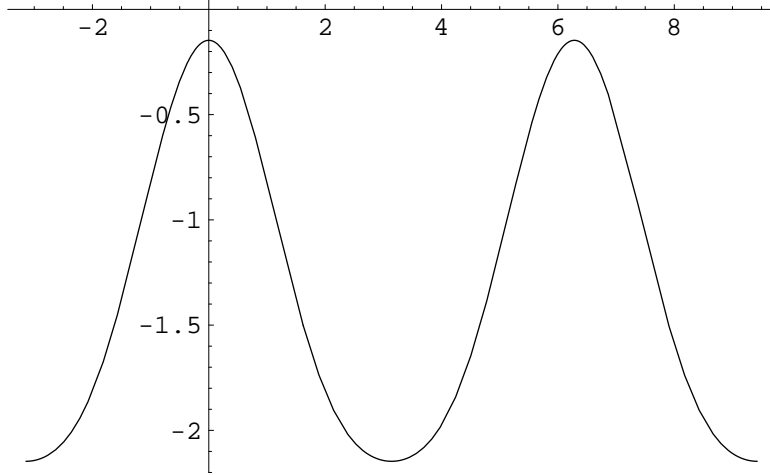


Figure 2: *The level 2 approximation of the tachyon profile for $\lambda = +1$*

with

$$\beta_n^3 = \langle \phi_{\pm n}, \Psi^{(3)} \rangle, \quad \phi_{\pm n} = e^{\pm i n X(0)} c \partial c(0) |0\rangle \quad (3.16)$$

By momentum conservation only the last line of (2.39) matters in the calculation of β_3^3 ,

which is given by

$$\begin{aligned}
\beta_3^3 &= \frac{1}{\sqrt{8}} \left\langle \phi_{-3}, \lim_{\Gamma \rightarrow \infty} \lim_{\Lambda \rightarrow \infty} \int_{e^{-\Lambda}}^1 dt_1 \int_{e^{-\Gamma}}^1 dt_2 \ cV^+(0)|0\rangle * U_{t_1}^\dagger U_{t_1}|0\rangle \right. \\
&\quad \left. * (B_L^+) cV^+(0)|0\rangle * U_{t_2}^\dagger U_{t_2}|0\rangle * (B_L^+) cV^+(0)|0\rangle \right\rangle \\
&= \frac{1}{\sqrt{8}} \left\langle \phi_3, \lim_{\Gamma \rightarrow \infty} \lim_{\Lambda \rightarrow \infty} \int_{e^{-\Lambda}}^1 dt_1 \int_{e^{-\Gamma}}^1 dt_2 \ cV^-(0)|0\rangle * U_{t_1}^\dagger U_{t_1}|0\rangle \right. \\
&\quad \left. * (B_L^+) cV^-(0)|0\rangle * U_{t_2}^\dagger U_{t_2}|0\rangle * (B_L^+) cV^-(0)|0\rangle \right\rangle. \tag{3.17}
\end{aligned}$$

All the operator involved have regular OPE and the result is that of [15] again.

$$\beta_3^3 = \frac{1}{\sqrt{8}} (2.148 \times 10^{-3}) \tag{3.18}$$

The calculation of β_1^3 is tedious but it is trivial, it receives a contribution from all the terms in (2.39). Next we list the contribution of each of them, where l_i stands for the contribution of the i^{th} line in (2.39).

$$l_1 = - \lim_{\Lambda \rightarrow \infty} \frac{\Lambda}{\sqrt{2}} \left[\frac{4}{3} \left(1 - \frac{3\sqrt{3}}{4\pi} \right) - 1 \right], \tag{3.19}$$

$$l_2 = \lim_{\Gamma \rightarrow \infty} \lim_{\Lambda \rightarrow \infty} \left[\frac{\Gamma}{\sqrt{2}} \left(-2e^\Lambda + \int_{e^{-\Lambda}}^1 dt \ f(t) \right) \right], \tag{3.20}$$

$$l_3 = - \lim_{\Gamma \rightarrow \infty} \lim_{\Lambda \rightarrow \infty} \frac{4e^\Lambda}{\sqrt{2}} \int_{e^{-\Gamma}}^1 dt_2 \ \frac{1}{t_2} \left\{ \frac{1}{t_2 + 2} \left[1 - \frac{2+t_2}{2\pi} \sin \left(\frac{2\pi}{2+t_2} \right) \right] \right\}, \tag{3.21}$$

$$l_4 = - \lim_{\Gamma \rightarrow \infty} \frac{4}{\sqrt{2}} \int_{e^{-\Gamma}}^1 dt_2 \ \frac{1}{t_2} \left\{ \frac{1}{t_2 + 2} \left[1 - \frac{2+t_2}{2\pi} \sin \left(\frac{2\pi}{2+t_2} \right) \right] - \frac{1}{4} \right\}, \tag{3.22}$$

$$l_5 = \frac{1}{\sqrt{8}} (0.734828) \tag{3.23}$$

$$\begin{aligned}
l_6 &= \lim_{\Gamma \rightarrow \infty} \lim_{\Lambda \rightarrow \infty} \frac{2\pi^2}{\sqrt{8}} \int_{e^{-\Lambda}}^1 dt_1 \int_{e^{-\Gamma}}^1 dt_2 \frac{t_2}{(2+l_2)^3} \left[1 - \frac{2+l_2}{2\pi} \sin \left(\frac{2\pi}{2+l_2} \right) \right] \\
&\quad \times \left[\sin^{-2} \left(\frac{\pi t_2}{2+l_2} \right) \sin^{-2} \left(\frac{\pi(1+t_2)}{2+l_2} \right) \sin^{-2} \left(\frac{2\pi}{2+l_2} \right) \sin^2 \left(\frac{\pi t_1 t_2}{2+l_2} \right) \sin^2 \left(\frac{\pi}{2+l_2} \right) \right. \\
&\quad + \sin^{-2} \left(\frac{\pi}{2+l_2} \right) \sin^{-2} \left(\frac{\pi t_2}{2+l_2} \right) \sin^{-2} \left(\frac{\pi t_1 t_2}{2+l_2} \right) \sin^2 \left(\frac{\pi(1+t_2)}{2+l_2} \right) \sin^2 \left(\frac{2\pi}{2+l_2} \right) \\
&\quad \left. + \sin^{-2} \left(\frac{\pi(1+t_2)}{2+l_2} \right) \sin^{-2} \left(\frac{\pi t_1 t_2}{2+l_2} \right) \sin^{-2} \left(\frac{2\pi}{2+l_2} \right) \sin^2 \left(\frac{\pi t_2}{2+l_2} \right) \sin^2 \left(\frac{\pi}{2+l_2} \right) \right] \tag{3.24}
\end{aligned}$$

$$\begin{aligned}
l_7 = & \lim_{\Gamma \rightarrow \infty} \lim_{\Lambda \rightarrow \infty} \frac{2\pi^2}{\sqrt{8}} \int_{e^{-\Lambda}}^1 dt_1 \int_{e^{-\Gamma}}^1 dt_2 \frac{t_2}{(2+l_2)^3} \left[1 - \frac{2+l_2}{2\pi} \sin\left(\frac{2\pi}{2+l_2}\right) \right] \\
& \times \left[\sin^{-2}\left(\frac{\pi t_1 t_2}{2+l_2}\right) \sin^{-2}\left(\frac{\pi(1+t_1 t_2)}{2+l_2}\right) \sin^{-2}\left(\frac{2\pi}{2+l_2}\right) \sin^2\left(\frac{\pi t_2}{2+l_2}\right) \sin^2\left(\frac{\pi}{2+l_2}\right) \right. \\
& + \sin^{-2}\left(\frac{\pi}{2+l_2}\right) \sin^{-2}\left(\frac{\pi t_1 t_2}{2+l_2}\right) \sin^{-2}\left(\frac{\pi t_2}{2+l_2}\right) \sin^2\left(\frac{\pi(1+t_1 t_2)}{2+l_2}\right) \sin^2\left(\frac{2\pi}{2+l_2}\right) \\
& \left. + \sin^{-2}\left(\frac{\pi(1+t_1 t_2)}{2+l_2}\right) \sin^{-2}\left(\frac{\pi t_2}{2+l_2}\right) \sin^{-2}\left(\frac{2\pi}{2+l_2}\right) \sin^2\left(\frac{\pi t_1 t_2}{2+l_2}\right) \sin^2\left(\frac{\pi}{2+l_2}\right) \right]
\end{aligned} \tag{3.25}$$

where $l_2 = t_2 + t_1 t_2$. Note that we have made a change of sign on the first five terms for the same reason we gave after equation (3.10). Each of these term can be evaluated numerically using mathematica and we finally obtain the following finite answer

$$\beta_1^3 = 0.798956. \tag{3.26}$$

With this we can write the level three approximation of the tachyon profile as

$$T(x) = -2.59791 \cos(x) + (0.15206) \cos(2x) - 1.29904 - 1.51887 \times 10^{-3} \cos(3x) \tag{3.27}$$

for $\lambda = -1$ and

$$T(x) = 2.59791 \cos(x) + (0.15206) \cos(2x) - 1.29904 + 1.51887 \times 10^{-3} \cos(3x) \tag{3.28}$$

for $\lambda = +1$.

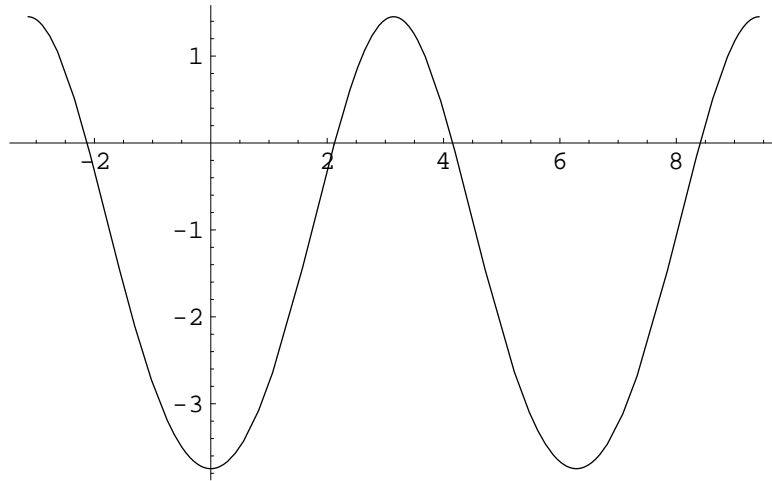


Figure 3: The level 3 approximation of the tachyon profile for $\lambda = -1$

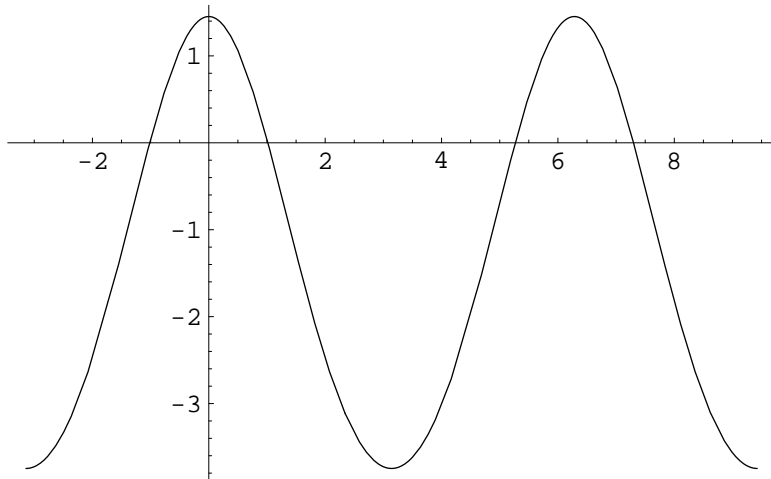


Figure 4: *The level 3 approximation of the tachyon profile for $\lambda = +1$*

At both level two and level three approximations, our result confirms the results from conformal field theory description that the $\cos(x)$ boundary deformation gives a solution representing a periodic array of D-branes placed at odd integral multiple of π when the coupling $\tilde{\lambda}$ is positive and at even integral multiple of π when the coupling $\tilde{\lambda}$ is negative. In both cases the D-brane is situated at the minimum of the interaction potential switched on along the boundary of the world-sheet. To first order approximation, the tachyon profile and this interaction potential can be identified. This means the first level approximation of the tachyon profile indicates the location of the D-branes. We just showed that including higher level contributions does not change the location of this minima and that means still with higher level contribution the tachyon profile minima is the location of the D-branes.

4. Conclusion

In this paper at the first place we could verify that an explicit expansion of the $\Phi^{(n)}$ in terms of definite L eigenvalue states, contains zero and negative eigenvalues only when the matter primary operator $V(z)$ has a singular OPE. This fact helped us to identify the terms which give rise to divergences in the case of singular OPE marginal deformations are those with zero or negative eigenvalues. As these kind of terms with the right ghost and twist number are very few, we conclude that one can determine exactly the form of the counter terms which have to be subtracted at any level of expansion in powers of λ to cancel the divergences associated with these terms. We have also seen that unlike the regular OPE case, where the entire solution satisfies the Schnabl gauge, only some piece of the solution can satisfy the Schnabl gauge in the case of singular OPE. We have shown this explicitly upto level 4 and it works the same for levels higher than that as the gauge violating terms of these levels are the same as those of the lowest levels.

In our computations we have considered only the case where the OPE is given by 2.15. However, as what matters is the commutation relation between the modes of $V(z)$, we

believe that the treatment in this paper can be generalized to any matter primary operator with arbitrary singular OPE and hence different commutation relation for the modes of $V(z)$.

In the second part of the paper we have considered the $\cos(x)$ marginal deformation, which from the world-sheet CFT point of view, is known to represent a periodic array of D-branes located at the minima of world-sheet potential. Using our results of the first part we could calculate the tachyon profile up to level 3 and obtained a result which agrees with the world-sheet description. Earlier, in the string field theory framework, the tachyon profile of a lump solution have been obtained in [36] using the level truncation method, when the transverse direction is compactified on a circle. Their result indicates that the lump solution represents a single D-brane placed at $x = 0$, which coincides with our solution if we restrict our solution to one period of the potential.

Lastly, we would like to comment on the depth of the minima of the tachyon profile which seems to increase as we go to higher and higher levels. As we mentioned before, to first order approximation, the tachyon profile is related to the world-sheet boundary interaction potential. This might lead one to the conclusion that the depth of the minima of the tachyon profile is related to the height of the potential. However, this can not be true since at each order approximation, our solutions are determined up to a Q_B closed additional terms, which if taken into account will affects the depth of the minima of the tachyon profile. Therefore, here we do not take the depth of the minima of the tachyon profile seriously, all we need is its position which is the position of the lower dimensional D-brane.

Acknowledgments

D.D.T. would like to thank Y. Okawa for his kind response to several questions I have asked. C. Park would like to thank the Isaac Newton Institute for Mathematical Sciences, where I was visting while this work was in progress, for their hospitality. This work is supported by the Science Research Center Program of the Korean Science and Engineering Foundation through the Center for Quantum SpaceTime (CQUeST) of Sogang University with grant number R11-2005-021.

References

- [1] M. Schnabl, *Analytic solution for tachyon condensation in open string field theory*, Adv. Theor. Math. Phys. **10** (2006) 433 [arXiv:hep-th/0511286].
- [2] Y. Okawa, *Comments on Schnabl's analytic solution for tachyon condensation in Witten's open string field theory*, JHEP **0604** (2006) 055 [arXiv:hep-th/0603159].
- [3] I. Ellwood and M. Schnabl, *Proof of vanishing cohomology at the tachyon vacuum*, JHEP **0702** (2007) 096 [arXiv:hep-th/0606142].
- [4] L. Rastelli and B. Zwiebach, *Solving open string field theory with special projectors*, arXiv:hep-th/0606131.
- [5] Y. Okawa, L. Rastelli and B. Zwiebach, *Analytic solutions for tachyon condensation with general projectors*, arXiv:hep-th/0611110.

- [6] E. Fuchs and M. Kroyter, *Universal regularization for string field theory*, JHEP **0702** (2007) 038 [arXiv:hep-th/0610298].
- [7] E. Fuchs and M. Kroyter, *On the validity of the solution of string field theory*, JHEP **0605** (2006) 006 [arXiv:hep-th/0603195].
- [8] E. Fuchs and M. Kroyter, *Schnabl's $L(0)$ operator in the continuous basis*, JHEP **0610** (2006) 067 [arXiv:hep-th/0605254].
- [9] L. Bonora, C. Maccaferri, R. J. Scherer Santos and D. D. Tolla, “Ghost story. I. Wedge states in the oscillator formalism,” arXiv:0706.1025 [hep-th].
- [10] H. Fuji, S. Nakayama and H. Suzuki, “Open string amplitudes in various gauges,” JHEP **0701**, 011 (2007) [arXiv:hep-th/0609047].
- [11] T. Erler, “Split string formalism and the closed string vacuum,” JHEP **0705**, 083 (2007) [arXiv:hep-th/0611200].
- [12] T. Erler, “Split string formalism and the closed string vacuum. II,” JHEP **0705**, 084 (2007) [arXiv:hep-th/0612050].
- [13] C. Imbimbo, “The spectrum of open string field theory at the stable tachyonic vacuum,” Nucl. Phys. B **770**, 155 (2007) [arXiv:hep-th/0611343].
- [14] M. Schnabl, *Comments on marginal deformations in open string field theory*, arXiv:hep-th/0701248.
- [15] M. Kiermaier, Y. Okawa, L. Rastelli and B. Zwiebach, *Analytic solutions for marginal deformations in open string field theory*, arXiv:hep-th/0701249.
- [16] E. Fuchs, M. Kroyter and R. Potting, *Marginal deformations in string field theory*, arXiv:0704.2222 [hep-th].
- [17] T. Erler, “Marginal Solutions for the Superstring,” JHEP **0707**, 050 (2007) [arXiv:0704.0930 [hep-th]].
- [18] Y. Okawa, “Analytic solutions for marginal deformations in open superstring field theory,” arXiv:0704.0936 [hep-th].
- [19] Y. Okawa, *Real analytic solutions for marginal deformations in open superstring field theory*, arXiv:0704.3612 [hep-th].
- [20] E. Fuchs and M. Kroyter, “Marginal deformation for the photon in superstring field theory,” arXiv:0706.0717 [hep-th].
- [21] M. Kiermaier and Y. Okawa, “Exact marginality in open string field theory: a general framework,” arXiv:0707.4472 [hep-th].
- [22] M. Kiermaier and Y. Okawa, “General marginal deformations in open superstring field theory,” arXiv:0708.3394 [hep-th].
- [23] A. Sen, “Descent relations among bosonic D-branes,” Int. J. Mod. Phys. A **14**, 4061 (1999) [arXiv:hep-th/9902105].
- [24] A. Recknagel and V. Schomerus, “Boundary deformation theory and moduli spaces of D-branes,” Nucl. Phys. B **545**, 233 (1999) [arXiv:hep-th/9811237].

- [25] C. G. . Callan, I. R. Klebanov, A. W. W. Ludwig and J. M. Maldacena, “Exact solution of a boundary conformal field theory,” Nucl. Phys. B **422**, 417 (1994)
- [26] J. Polchinski and L. Thorlacius, “Free fermion representation of a boundary conformal field theory,” Phys. Rev. D **50**, 622 (1994) [arXiv:hep-th/9404008].
[arXiv:hep-th/9402113].
- [27] C. G. . Callan and I. R. Klebanov, “Exact $C = 1$ boundary conformal field theories,” Phys. Rev. Lett. **72**, 1968 (1994) [arXiv:hep-th/9311092].
- [28] H. Kogetsu and S. Teraguchi, “Massless boundary sine-Gordon model coupled to external fields,” JHEP **0501**, 048 (2005) [arXiv:hep-th/0410197].
- [29] A. Sen, “On the background independence of string field theory,” Nucl. Phys. B **345**, 551 (1990).
- [30] A. Sen and B. Zwiebach, “Large marginal deformations in string field theory,” JHEP **0010**, 009 (2000) [arXiv:hep-th/0007153].
- [31] T. Takahashi and S. Tanimoto, “Marginal and scalar solutions in cubic open string field theory,” JHEP **0203**, 033 (2002) [arXiv:hep-th/0202133].
- [32] J. Kluson, “Exact solutions in open bosonic string field theory and marginal deformation in CFT,” Int. J. Mod. Phys. A **19**, 4695 (2004) [arXiv:hep-th/0209255].
- [33] J. Kluson, “Exact solutions in SFT and marginal deformation in BCFT,” JHEP **0312**, 050 (2003) [arXiv:hep-th/0303199].
- [34] A. Sen, “Energy momentum tensor and marginal deformations in open string field theory,” JHEP **0408**, 034 (2004) [arXiv:hep-th/0403200].
- [35] A. Sen, “Rolling tachyon,” JHEP **0204**, 048 (2002) [arXiv:hep-th/0203211].
- [36] N. Moeller, A. Sen and B. Zwiebach, “D-branes as tachyon lumps in string field theory,” JHEP **0008**, 039 (2000) [arXiv:hep-th/0005036].